Externalities - A numerical example

Manuel Pacheco Coelho

2020/21

Externalities- A numerical example *

Suppose an Airport that is located near an area where a Constructor is developping its activities.

Be:

C- the number of constructed buildings

A- the number of planes in transit by this airport.

The profit function of the airport is

$$\Pi_A = 40A - A^2$$

The profit of the constructor depends not only of the number of constructed houses but also of traffic intensity (the price of houses falls when the noise of the planes is growing)

The profit function of the Constructor is, then:

$$\Pi_C = 55C - C^2 - AC$$

- i) Determine C, A, $\Pi_{A,}$, Π_C (suppose that the decisions of the two agents are independent – pure competition)

- For the entity that manages the airport the decision process is:
- Max $\Pi_{A,} = 40A - A^2$
- $d\Pi_{A,} /dA = 40 – 2A = 0$ ▫ **A =20**

- The profit is $\Pi_{A,} = 40 (20) – 20^2 = 400$

  $\Pi_A = 400$

- The behaviour of the constructor:

- Max Π_C / A = 20
- d Π_C /dC = 55 – 2C –A = 0,   As A= 20

  55 – 2C – 20 = 0   **C= 17,5**

Π_C = 306,25

- ii) The Government decides to “close the airport”. What are the changes in the results?

- The airport activity would be zero.
- **A = 0**
- $\mathbf{\Pi_{A,} = 0}$

- The Max of $\mathbf{\Pi_C}$ turns
- $d\,\Pi_C / dC = 55 - 2C = 0$ **C = 27,5**

- $\Pi_C = 55\,(27,5) - 27,5^2 - 0\,(27,5)$

  $\mathbf{\Pi_C = 756,25}$

- <u>Total Profits</u>

- First situation
- $\Pi_T = \Pi_A + \Pi_C = 400 + 306{,}25 = 706{,}25$

- Second situation
- $\Pi_T = 0 + 756{,}25 = 756{,}25$

- Total profit is higher in the second >>> this policy conducts to a better position in terms of welfare.

- iii) The Government pretends to introduce a Tax (Pigouvian Tax) equal to the damage created to the Constructor by the airport traffic.

- External Costs are   CE = AC
- Marginal External Costs CMg E = dCE/dA = C
- CMgE = C

- Cmg P < Cmg S
- CMg P + CMg E = CMg S

- The Pigouvian Tax should be equal to the marginal external cost at the optimal solution
- T = CMg E   >>>>>>> **T = C**

- Tax is applied to the airport >>> the profit function turns:
- $\Pi_A = 40A – A^2 – TA$

  $\Pi_A = 40A – A^2 – CA$
- The other profit function maintains:
- $\Pi_C = 55C – C^2 – AC$

- The Calculus of the value of T comes from the resolution of the System

  $d\,\Pi_A /dA$

  $d\,\Pi_C /dC$

$d\,\Pi_A /dA = 40 – 2A – C = 0$

$d\,\Pi_C /dC = 55 – 2C – A = 0$

- **C (=T) = 70/3**
- **A = 25/3**

Profits?

- $\Pi_A{}^{'} = 40\,(25/3) - (25/3)^2 - (70/3)(25/3) = 69{,}4$
- $\Pi_C{}^{'} = 544{,}(4)$

Taxes Collected by the Government

- $T_T = A * T$ = Traffic * Tax by Plane
  $= AC = 194{,}4$

Social Surplus (welfare)

- $W = \Pi_A{}^{'} + \Pi_C{}^{'} + T_T =$
- 69,44 + 544,(4) + 194,4 **= 808,3**

- iv) Suppose a fusion between the two firms: Results? Compare iii) and iv).

- Seems the cartel situation.
- Used to study "Rights Based Management"; The approach of COASE.

- Profit agregate function:

$$\Pi_{T(AG)} = \Pi_A + \Pi_C = (40A - A^2) + (55C - C^2 - AC)$$

The "Sole Owner" chooses A and C that maximizes the global/total profit >>>>

- Solving the System

  $d\,\Pi_T/dA = 0$

  $d\,\Pi_T/dC = 0$

$d\,\Pi_T/dA = 40 - 2A - C = 0$ **$C=70/3$**

$d\,\Pi_T/dC = 55 - 2C - A = 0$ **$A = 8,(3)$**

$\Pi_T = 808,(3)$

Note: The result is the same of the Pigouvian tax >>> In terms of efficiency the result is equal.

But the the distribution of the welfare is different. In the case of PIGOU policy there is a third agent in the redistribution of the social surplus (the Government); in the case of COASE approach only the private agents appropriate the surplus.

- **Nota:**

  Este Exercício fazia parte, originalmente, do Caderno de Exercícios da UL de MICROECONOMIA II , da Licenciatura em Economia do ISEG/UTL.